

AMATYC - Spring '11

① $0.8E = 2.06A \rightarrow E = 1.5A \rightarrow \text{Answer: } \textcircled{D} = 150$

② $a \# b = a(b^2 + 1)$
 $(a \# b) \# 3 = a(b^2 + 1)(3^2 + 1) = 250 \rightarrow a(b^2 + 1) = 25$
 $\rightarrow b^2 + 1 = 5$ (integer $> 1, < 25$, square + 1)
 $\rightarrow b = 2$
 $\rightarrow a = 5$ } $a \# b = 7$ $\textcircled{B} = 7$

③ Let C_n denote # of possible ways to climb n steps. She can get there brushing with step of length 1, 2 or 4. So:

$$C_n = C_{n-1} + C_{n-2} + C_{n-4}$$

(10) No 2's allowed after place 2 (appropriate 3-cut makes it composite).
 The lowest we can go (to minimize) is pattern=1131131 because 131, 311
 are both prime. Answer: 1131131131 Answer: 1131131

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$$(17) \quad d \mid n^2 + 7n + 4 \rightarrow d \mid n^2 + 7(n+4) + (-4) \rightarrow d \mid n^2 + 7 - (n+4)^2 \rightarrow d \mid 23$$