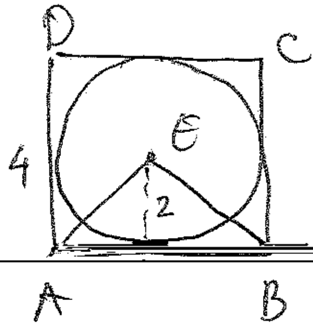
AMATYC – Spring 2010

1) $P(4) = 32+8 = 40$, $P(2) = 2$
$P(4) - P(2) = 38 = 2 \cdot 19$ B : 19

2) 
$\frac{1}{4}(A_{SQ} - A_{CIR}) = \frac{1}{4}(16 - 4\pi) = 4 - \pi$ C : $4 - \pi$

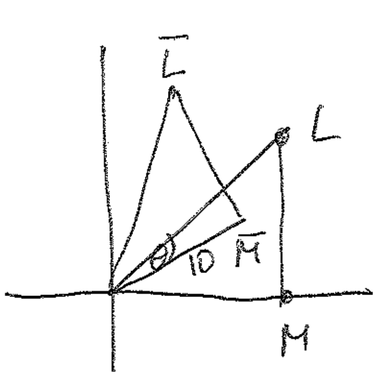
3) $2a + b = 10$
$3b + a = 8 \rightarrow a = 8 - 3b$
$16 - 5b = 10$
$b = 1.2$, $a = \ldots$
$a + b = 5.6$ B : 5.6

4) $\frac{x+1}{x-3} - 2 \geq 0 \rightarrow \frac{x+1-2x+6}{x-3} \geq 0$
$\rightarrow \frac{7-x}{x-3} \geq 0$
$- \quad (3) \quad + + + \quad (7] \quad -$ E : $(3, 7]$

5) $2 + 3d = (2+d)^2 - 8$
$d^2 + d - 6 = 0$ [since $d > -1$] $\Rightarrow d = 2$
$a_3 = 2 + 3d = 8$ C : 8

(10) 112, 113, 115, 121, 131, 151, 211, 311, 311

(11)



Overlaping triange

$A = \frac{10 \cdot 10 \tan 15^\circ}{2} = 50 \tan 15^\circ$

$A = \frac{10 \cdot 10}{2} \cdot 2 - 50 \tan 15^\circ \approx 86.6$

E: 87

(12)

$ab = 48$
$ac = 50$
$bc = 54$

$V = abc = \sqrt{48 \cdot 50 \cdot 54} = 360$

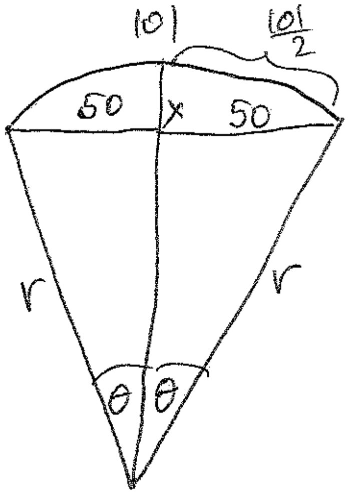
A: 360

(13)

$2010 = 2 \cdot 3 \cdot 5 \cdot 67$

Every factor of 2010 has form: $2^{k_1} 3^{k_2} 5^{k_3} 67^{k_4}$ wher k_i's are either 0's or 1's. There are 16 of them. 8 of them have $k_4 = 1$, 8 of them $k_4 = 0$ and so on. When we multiply them we get $2^8 \cdot 3^8 \cdot 5^8 \cdot 67^8$. This is product of all 4 columns.

16



$r\theta = \frac{101}{2}$

$r\sin\theta = 50$

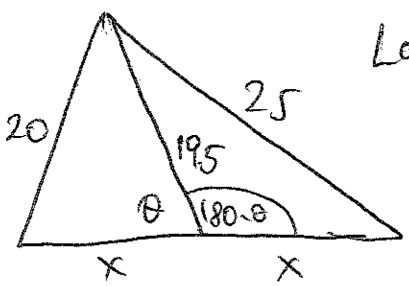
$\Big\} \quad \frac{\sin\theta}{\theta} = \frac{100}{101} \xrightarrow{\text{calculator}} \theta \approx 0.244$

$r \approx 207$

$x = 207 - 207\cos\theta \approx 6.13$

D: 6

17



Law of cosines: $20^2 = x^2 + 19.5^2 - 39x\cos\theta$

$25^2 = x^2 + 19.5^2 + 39x\cos\theta$

Add: $20^2 + 25^2 = 2x^2 + 2\cdot 19.5^2$

$x = \frac{1}{2}\sqrt{400 + 625 - 2\cdot 19.5^2} = 11.5$

$2x = 23$ B: 23

18 $abba = 1001a + 110b \equiv_7 5b$ because $1001 = 7\cdot 143$, $110 = 7\cdot 15 + 5$